$S * - : sSet \longrightarrow sSet_{S/}$ and $- * T : sSet \longrightarrow sSet_{T/}$

$$sSet_{S/} \longrightarrow sSet \qquad\qquad sSet_{T/} \longrightarrow sSet$$

$$(p : S \to X) \longmapsto X_{p/} \qquad\qquad (q : T \to X) \longmapsto X_{/q}$$

slice under p slice over q

<u>Universal Properties</u>:

$$\left\{ \begin{tikzcd} S \arrow[r, "p"] \arrow[d, hook] & X \\ S * K \arrow[ur, dashed] & \end{tikzcd} \right\} \overset{\sim}{\leftrightarrows} \left\{ K \dashrightarrow X_{p/} \right\} \qquad \left\{ \begin{tikzcd} T \arrow[r, "q"] \arrow[d, hook] & X \\ K * T \arrow[ur, dashed] & \end{tikzcd} \right\} \leftarrow \left\{ K \dashrightarrow X_{/q} \right\}$$

• $\mathrm{Hom}_{sSet_{S/}}(S * K, X) \simeq \mathrm{Hom}_{sSet}(K, X_{p/})$

$K = \Delta^n \overset{\text{formula}}{\rightsquigarrow} X_{p/} \qquad X_{/q}$

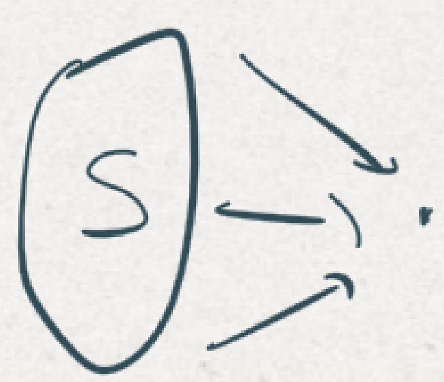
$$\begin{tikzcd} & S \arrow[dl] \arrow[dr] & \\ {} & & {} \end{tikzcd}$$

• $(X_{p/})_n = \mathrm{Hom}_{sSet_{S/}}(S * \Delta^n, X)$

$n = 0$

$$\begin{tikzcd} S * \Delta^0 \arrow[r] \arrow[d, squiggly] & X \\ S^{\triangleright} & S \arrow[ul] \arrow[u, "p"'] \end{tikzcd}$$

$p(s) \longrightarrow x$



Example: $p : \Delta^0 \xrightarrow{x} X$

$$\begin{tikzcd} \mathrm{Hom}_{sSet}(K, X_{x/}) \simeq \mathrm{Hom}_{sSet_{\Delta^0/}}((K^{\triangleright}, v), (X, x)) \arrow[d] \\ sSet_* \end{tikzcd}$$

<u>Slice as a functor</u>

$\mathrm{Fun}(-,-) : sSet^{op} \times sSet \longrightarrow sSet$

Sort of true for the slice, but we have to consider this category of <u>twisted arrows</u>,

$$\mathrm{Tw}(\mathcal{C}) \rightsquigarrow \begin{cases} \text{ob: } \mathrm{Ar}(\mathcal{C})_0 \quad f: S \to X \\ \text{Mor: } \end{cases}$$

$$\begin{tikzcd} S \arrow[r, "f"] & X \arrow[d, "p"] \\ T \arrow[u, "j"] \arrow[r, "pfj"'] & Y \end{tikzcd}$$

$$T \xrightarrow{j} S \xrightarrow{f} X \xrightarrow{p} Y \rightsquigarrow \text{"restriction" map}$$

$$X_{f/} \longrightarrow Y_{pfj/}$$

$$\begin{tikzcd} & S \arrow[dl] \arrow[d, "f"] \\ S * \Delta^n \arrow[r, "\eta"'] & X \end{tikzcd} \longmapsto \begin{tikzcd} T \arrow[d] \arrow[rrr, bend left, "pfj"] & & & \\ T * \Delta^n \arrow[r] & S * \Delta^n \arrow[r, "\eta"'] & X \arrow[r, "p"'] & Y \end{tikzcd}$$

Special Cases:

"Restriction maps" $f : S \to X$

$$X_{f/} \longrightarrow X$$

$$\boxed{\emptyset \hookrightarrow S \xrightarrow{f} X \xrightarrow{p} Y} \in Tw(sSet)_1$$

$$\begin{tikzcd} S \arrow[r, "f"] & X \arrow[d] \\ \emptyset \arrow[u, hook] \arrow[r] & X \end{tikzcd}$$

$$X_{\emptyset \hookrightarrow X/} = X$$

$$\begin{tikzcd} & S \arrow[dl] \arrow[dr, "f"] & \\ \Delta^n * S \arrow[rr, "x"'] & & X \end{tikzcd} \rightsquigarrow \begin{tikzcd} \Delta^n \simeq \Delta^n * \emptyset \arrow[r, hook] \arrow[rr, bend right, "\tilde{x}"'] & \Delta^n * S \arrow[r] & X \end{tikzcd}$$

$$X_{f/} \longrightarrow Y_1$$

$$\begin{tikzcd} & x \arrow[dl, "\eta_s"'] \arrow[dr, "\eta_{s'}"] & \\ f(s) \arrow[r] \arrow[d] & & f(s') \arrow[d] \\ p(f(s)) \arrow[rr] & & p(f(s)) \end{tikzcd}$$

$$\emptyset \hookrightarrow S \to X \xrightarrow{p} Y$$

Prop: Let $\mathcal{C}$ be an ∞-category. Pick $x \in \mathcal{C}_0$.

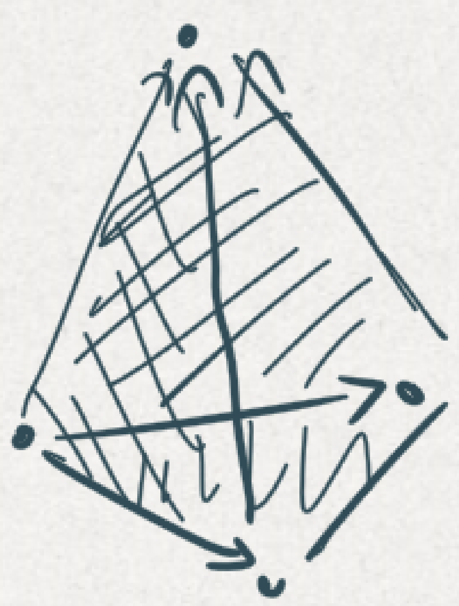
$\mathcal{C}_{/x} \longrightarrow \mathcal{C}$ is a right fibration. $\Rightarrow$ $\mathcal{C}_{x/}$ is an ∞-category

$$\begin{tikzcd} \Lambda^n_j \arrow[r] \arrow[d, hook] & X \arrow[d, "p"] \\ \Delta^n \arrow[r] \arrow[ur, dotted] & Y \end{tikzcd} \rightsquigarrow$$

LFib $\cap$ RFib = KanFib

$$\begin{tikzcd} \Lambda^n_j \arrow[r, "f"] \arrow[d, hook] & \mathcal{C}/x \arrow[d] \\ \Delta^n \arrow[r, "g"'] \arrow[ur, dotted] & \mathcal{C} \end{tikzcd} \Longleftarrow$$

$$\begin{tikzcd} \phi * \Delta^0 \arrow[r, hook] & (\Lambda^n_j * \Delta^0) \cup_{\Lambda^n_j * \phi} (\Delta^n * \phi) \arrow[r, "(\tilde{f}, g)"] \arrow[d] & \mathcal{C} \\ & \Delta^n * \Delta^0 \arrow[ur, dotted] & \end{tikzcd}$$



$$\begin{tikzcd} \{n+1\} \arrow[r, hook] & \Lambda^{n+1}_j \arrow[r] \arrow[d, hook] & \mathcal{C} \\ & \Delta^{n+1} \arrow[ur, dotted] & \end{tikzcd}$$

$$\Lambda^n_j = \bigcup_{k \in [n] \setminus j} \Delta^{[n]\setminus k} * \Delta^0$$

$$\begin{tikzcd} K*\{0\} \arrow[r, hook] \arrow[rr, bend left, "\hat{f}"] & K*\partial\Delta^n \arrow[r] \arrow[d, hook] & \mathcal{C} \\ & K*\Delta^n \arrow[ur, dashed] & \end{tikzcd}$$

$x \in \mathcal{C}_0$

$(\mathcal{C}_{f/})$

$f : S \longrightarrow \mathcal{C}$